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A COMPOSITE ESTIMATOR FOR SMALL DOMAINS AND ITS SENSITIVITY INTERVAL FOR WEIGHTS α

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ABSTRACT

The composite estimation methods, suggested in the literature, have major problems to deal with the estimation of optimum weights to combine synthetic and direct estimators. To take care of the absence of optimum weights, we have obtained the sensible interval of involved weights in the form of better performance interval of the weights with a view to retaining superiority for the different composite estimators i.e. for the direct ratio vs. synthetic ratio composite estimator and simple direct vs. synthetic ratio composite estimator.

Key words: Small area (domain) estimation, Synthetic and Composite estimation, Proportional inflation, TRS.

1. Introduction

Tikkiwal, G.C. and Ghiya (2000) define a generalized class of synthetic estimators, using auxiliary information. The proposed class of synthetic estimators gives consistent estimators if the corresponding synthetic assumptions are satisfied. The generalized class, among others includes the simple synthetic, ratio synthetic and product synthetic estimators. Further ,the generalized synthetic estimator is used to construct a generalized class of composite estimators by combining with generalized direct ratio estimators. Generalized class, among others include simple ratio, ratio synthetic, composite estimator taking combination of various direct and synthetic estimators. These authors, further, discuss the generalized class of synthetic and composite estimators under SRS and stratified random sampling schemes. It has been shown by Tikkiwal, G.C. and Ghiya (2004), Tikkiwal, G.C. and Pandey (2007) that, when auxiliary variable is closely related with variable under study, small area estimators based

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on auxiliary information perform better than those which do not use auxiliary information. Further, Tikkiwal, G.C. and Pandey (2007) discuss the generalized class of synthetic and composite estimators under Lahiri-Midzuno and systematic sampling schemes. The relative performance of these estimators is empirically assessed through a simulation study for the problem of crop acreage estimation for small domains. They recommend the use of composite estimator, which is a weighted sum of direct ratio and ratio synthetic estimators, for crop acreage estimation for small domains under the TRS (Timely Reporting Scheme), for such case where the synthetic assumption is satisfied. Most of the times such sampling schemes are only of theoretical interest, as in large-scale sample surveys mostly multistage sampling designs are used.

2. Composite Estimator and its sensitivity Interval for Weights α

Let us consider the case of composite estimator as direct ratio estimator shrinkage with ratio synthetic estimator, i.e. denote

$$\bar{y}_{c,a} = \alpha\, \bar{y}_{d,r,a} + (1-\alpha)\, \bar{y}_{syn,r,a} \tag{2.1}$$

Here α is the weight assign to them and "a" denote domain. Let us denote 'P' as the proportional inflation in the variance of $\bar{y}_{c,a}$ resulting from the use of some α other than α_{opt} i.e.

$$P = \frac{MSE(\bar{y}_{c,a}) - MSE_{opt}(\bar{y}_{c,a})}{MSE_{opt}(\bar{y}_{c,a})} \tag{2.2}$$

Now, to obtain P we have to compute MSE of composite estimator under α and α_{opt} . Thus, expression for P is given as

$$P = \frac{(\alpha^2 - \alpha_{opt}^2) MSE(\bar{y}_{d,r,a}) + \{(1-\alpha)^2 - (1-\alpha_{opt})^2\} MSE(\bar{y}_{syn,r,a})}{\alpha_{opt}^2 MSE(\bar{y}_{d,r,a}) + (1-\alpha_{opt})^2 MSE(\bar{y}_{syn,r,a})}$$

$$= \frac{\left(\frac{1-\alpha}{1-\alpha_{opt}}\right)^2 \left[\left(\frac{\alpha}{(1-\alpha)}\right)^2 MSE(\bar{y}_{d,r,a}) + MSE(\bar{y}_{syn,r,a})\right] - \left[\left(\frac{\alpha_{opt}}{(1-\alpha_{opt})}\right)^2 MSE(\bar{y}_{d,r,a}) + MSE(\bar{y}_{syn,r,a})\right]}{\left(\frac{\alpha_{opt}}{1-\alpha_{opt}}\right)^2 MSE(\bar{y}_{d,r,a}) + MSE(\bar{y}_{syn,r,a})}$$

$$= \left[\frac{1-\alpha}{1-\alpha_{opt}}\right]^2 P_1 \quad -1 \tag{2.3}$$

where P_1 is given as

$$P_1 = \frac{\left(\frac{\alpha}{1-\alpha}\right)^2 MSE(\bar{y}_{d,r,a}) + MSE(\bar{y}_{syn,r,a})}{\left(\frac{\alpha_{opt}}{1-\alpha_{opt}}\right)^2 MSE(\bar{y}_{d,r,a}) + MSE(\bar{y}_{syn,r,a})}$$

Now, equation (2.2) is positive and gives

$$P_1 \geq \left[\frac{1-\alpha_{opt}}{1-\alpha}\right]^2 \tag{2.4}$$

$$MSE(\bar{y}_{d,r,a}) = \left(\frac{N-n}{N\,n}\right)\left[\frac{N_a - 1}{N-1}\right] S^2_{EUa}$$

where

$$S^2_{EUa} = \frac{1}{N_a - 1}\sum_{U_a}(y_k - B_a x_k)^2$$

with $$B_a = \frac{(\sum_{U_a} y_k)}{(\sum_{U_a} x_k)}$$

$$MSE(\bar{y}_{syn,r,a}) = \left(\frac{\bar{Y}}{\bar{X}}\bar{X}_a\right)^2 \left[1 + \frac{N-n}{N\,n}\left\{3\,C_x^2 + C_y^2 - 4C_{xy}\right\}\right]$$

$$-\,2\,\bar{Y}_a\left(\frac{\bar{Y}}{\bar{X}}\bar{X}_a\right)\left[1 + \frac{N-n}{N\,n}\left\{\,C_x^2 - C_{xy}\right\}\right] + \bar{Y}_a^2$$

If synthetic assumption is satisfied then

$$MSE(\overline{y}_{syn,r,a}) = \frac{N-n}{N\,n}\left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2$$

Thus

$$P_1 = \frac{\left(\frac{\alpha}{1-\alpha}\right)^2 MSE(\overline{y}_{d,r,a}) + MSE(\overline{y}_{syn,r,a})}{\left(\frac{\alpha_{opt}}{1-\alpha_{opt}}\right)^2 MSE(\overline{y}_{d,r,a}) + MSE(\overline{y}_{syn,r,a})}$$

$$= \frac{\left(\frac{\alpha}{1-\alpha}\right)^2 \left(\frac{N_a-1}{N-1}\right) S_{EUa}^2 + \left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2}{\left(\frac{\alpha_{opt}}{1-\alpha_{opt}}\right)^2 \left(\frac{N_a-1}{N-1}\right) S_{EUa}^2 + \left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2}$$

From equation (2.4) we have

$$\frac{\left(\frac{\alpha}{1-\alpha}\right)^2 \left(\frac{N_a-1}{N-1}\right) S_{EUa}^2 + \left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2}{\left(\frac{\alpha_{opt}}{1-\alpha_{opt}}\right)^2 \left(\frac{N_a-1}{N-1}\right) S_{EUa}^2 + \left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2} \geq \frac{\left(1-\alpha_{opt}\right)^2}{\left(1-\alpha\right)^2}$$

so

$$\alpha^2 \left(\frac{N_a-1}{N-1}\right) S_{EUa}^2 + \left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2 (1-\alpha)^2 \geq \alpha_{opt}^2 \left(\frac{N_a-1}{N-1}\right) S_{EUa}^2$$
$$+\left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2 (1-\alpha_{opt})^2$$

$$\Rightarrow (\alpha^2 - \alpha_{opt}^2)\left(\frac{N_a-1}{N-1}\right) S_{EUa}^2 \geq \left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2 \left\{(1-\alpha_{opt})^2 - (1-\alpha)^2\right\}$$

$$\Rightarrow (\alpha + \alpha_{opt})\left(\left(\frac{N_a-1}{N-1}\right) S_{EUa}^2 + \left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2\right) \geq 2\left(C_x^2 + C_y^2 - 2\,C_{xy}\right)\overline{Y}_a^2$$

$$\alpha \geq 2\frac{\left(C_x^2+C_y^2-C_{xy}\right)\bar{Y}_a^2}{\left(\frac{N_a-1}{N-1}\right)S_{EUa}^2+\left(C_x^2+C_y^2-C_{xy}\right)\bar{Y}_a^2}-\alpha_{opt}$$

or

$$\left(\alpha+\alpha_{opt}\right)\geq 2\frac{\left(C_x^2+C_y^2-C_{xy}\right)\bar{Y}_a^2}{\left(\frac{N_a-1}{N-1}\right)S_{EUa}^2+\left(C_x^2+C_y^2-C_{xy}\right)\bar{Y}_a^2} \tag{2.5}$$

Now, let us consider the case when we have composite estimator of the form simple direct and synthetic ratio, i.e.

$$\bar{y}_{c,\,a}=\alpha\;\bar{y}_{d,\;a}+(1-\alpha)\;\bar{y}_{syn,\,r,\,a} \tag{2.6}$$

Here

$$MSE(\bar{y}_{d,\,a})=\frac{N-n}{N\;n}\left(\frac{S_a^2}{\phi_a}\right)$$

where $\phi_a=\dfrac{N_a}{N}$ and $S_a^2=\dfrac{1}{N_a-1}\sum_{i\in a}(y_i-\bar{Y}_a)^2$

Using (2.4) we have

$$\frac{\left(\frac{\alpha}{1-\alpha}\right)^2\frac{S_a^2}{\phi_a}+\left(C_x^2+C_y^2-2\;C_{xy}\right)\bar{Y}_a^2}{\left(\frac{\alpha_{opt}}{1-\alpha_{opt}}\right)^2\frac{S_a^2}{\phi_a}+\left(C_x^2+C_y^2-2\;C_{xy}\right)\bar{Y}_a^2}\geq\frac{\left(1-\alpha_{opt}\right)^2}{\left(1-\alpha\right)^2}$$

So

$$\alpha^2\frac{S_a^2}{\phi_a}+\left(C_x^2+C_y^2-2\;C_{xy}\right)\bar{Y}_a^2\left(1-\alpha\right)^2\geq\alpha_{opt}^2\frac{S_a^2}{\phi_a}+\left(C_x^2+C_y^2-2\;C_{xy}\right)\bar{Y}_a^2(1-\alpha_{opt})^2$$

$$\Rightarrow(\alpha^2-\alpha_{opt}^2)\frac{S_a^2}{\phi_a}\geq\left(C_x^2+C_y^2-2\;C_{xy}\right)\bar{Y}_a^2\left\{(1-\alpha_{opt})^2-\left(1-\alpha\right)^2\right\}$$

$$\Rightarrow (\alpha+\alpha_{opt})\left(\frac{S_a^2}{\phi_a}+\left(C_x^2+C_y^2-2\,C_{xy}\right)\overline{Y}_a^2\right)\geq 2\left(C_x^2+C_y^2-2\,C_{xy}\right)\overline{Y}_a^2$$

$$\Rightarrow \alpha \geq 2\frac{\left(C_x^2+C_y^2-C_{xy}\right)\overline{Y}_a^2}{\frac{S_a^2}{\phi_a}+\left(C_x^2+C_y^2-C_{xy}\right)\overline{Y}_a^2}-\alpha_{opt}$$

or

$$\left(\alpha+\alpha_{opt}\right)\geq 2\frac{\left(C_x^2+C_y^2-C_{xy}\right)\overline{Y}_a^2}{\frac{S_a^2}{\phi_a}+\left(C_x^2+C_y^2-C_{xy}\right)\overline{Y}_a^2} \tag{2.7}$$

Conclusion

In either cases when α takes values other than α_{opt}, from (2.5) and (2.7) we have the range of weights for the proposed composite estimators which provide P value positive i.e. $P=\dfrac{MSE(\overline{y}_{c,\,a})-MSE_{opt}(\overline{y}_{c,\,a})}{MSE_{opt}(\overline{y}_{c,\,a})}>0$

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